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A comparative analysis of marketing promotions and implications for data analytics

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ABSTRACT

Retailers use many different marketing promotions to increase sales and profits. These promotions include price reductions, coupons, cash mail-in rebates, free gift cards, and buy-one-get-one (BOGO) discounts. The type of promotion used results in different outcomes for demand, profit, average price, consumer surplus, and sales taxes collected. We perform comparative analysis of these five promotions and their outcomes. We show that for the same discount amount, price reductions result in the lowest average price. For products with weakly diminishing consumer utility and low consumer stockpiling, BOGO promotions result in the largest demand, profit, consumer surplus, and taxes collected. Cash mail-in rebates may result in large profit and taxes collected, but they perform poorly in terms of average price paid and consumer surplus. We also find that a retailer offering a delayed incentive (i.e. gift cards and mail-in rebates) offers a larger reward but provides lower consumer surplus than when offering an immediate incentive (i.e. price reduction and BOGO). In a segmented market with a price-insensitive consumer segment, immediate incentives have the disadvantage of allowing price-insensitive consumers arriving during the promotion to obtain the discount, which reduces the discount effectiveness. The addition of more retailer objectives to maximizing profit, such as demand maximization or consumer surplus, increases the effectiveness of immediate incentives. We also provide a framework for estimating the important parameters for evaluating promotion effectiveness using readily available transactional data and examine its accuracy using a simulation experiment.

1. Introduction

Retailers use many types of sales promotions. A consumer shopping at a large retailer is likely to see signs advertising "Take 30% off regular price" on some products, "buy one at the regular price get the second for $\frac{1}{2}$ off" on other products, or find ads containing coupons entitling her to take some amounts off the regular products' prices. These incentives are enjoyed immediately whereas other types of promotions provide a delayed incentive. Signs displaying "\$20 cash mail-in rebate" or "Buy one unit and get a free \$5 gift card" can be seen at many retailers. These savings are enjoyed only by those consumers who buy the product and put forth the effort to get the reward, i.e. successfully complete the rebate redemption process or redeem the gift card during the specified valid period.

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The above promotions are usually offered in combination with advertising and/or special displays to maximize their impact (Blattberg & Neslin, 1989). This makes it difficult to measure the true effect of the monetary promotion. Furthermore, these promotions frequently induce consumers to stockpile the product (Wansink & Deshpande, 1994). If consumers' overall consumption rate of the product increases significantly, then future demand of the product is unaffected. On the other hand, if the increase in the consumption rate is small, then a post-promotion dip in demand will occur. Findings from research on the existence and magnitude of this dip have been inconclusive but point in the direction of this dip being small (Hendel & Nevo, 2003).

There is much research investigating price reduction promotions (Raghubir, 2004) and coupons (Gupta & Cooper, 1992). Much less research investigating cash mail-in rebates (Soman, 1998), free gift cards (Khouja, Pan, Ratchford, & Zhou, 2011), and buy-one-get-one (Sinha & Smith, 2000) can be found in the literature. Some research has focused on comparing two types of promotions such as coupons vs. rebates (Lu & Moorthy, 2007) and enumerating research issues in need of resolution (Blattberg & Neslin, 1989). This research does not reflect the retailing landscape. For example, the Black Friday ad of Kohl's for 11/19/2018–11/23/2018 had 135 products with free gift cards (FGC) compared to 501 products with price reduction. The Publix grocery chain ad for 11/14/2018–11/21/2018 had 60 buy-one-get-one (BOGO) offers compared to 86 price reductions. Therefore, it is clear that further investigation and comparison of these promotions are needed.

The purpose of this paper is to develop a set of measures for evaluating monetary promotions offered by retailers from consumers', retailers', and government's perspectives. In doing so, we identify important parameters needed to evaluate monetary promotions and a methodology for estimating these parameters. Our focus is on the "impact on practice" (Lilien, Roberts, & Shankar, 2013) in the sense that we do not try to describe the mental accounting of consumer behavior which leads to a given estimate of a parameter. Rather we focus on obtaining accurate estimates of the needed parameters using historical data. For example, we do not analyze a consumer's decision to incorporate coupons into price computation but we focus on estimating that probability from historical data.

Our analysis excludes purchase-amount specific promotions. These promotions reward consumers for purchasing any set of products exceeding a specified $ amount threshold. Retailers may use coupons such as World Market's frequently offered promotion "save $10 on your next purchase of $50 or more" or Harris Teeters's "$10 off any $50 grocery purchase". Retailers also use free gift cards/gift vouchers such as Kohl's "get $10 Kohl's cash for every $50 you spend" and Target's "free $10 Target gift card with food and/or drink purchase of $50 or more".

The rest of the paper is organized as follows. In Section 2, we describe product-specific promotions. In Section 3, we identify the parameters needed to measure the effectiveness of different promotions, develop a set of comparative effectiveness measures from retailers', consumers', and government's perspective, and conduct a comparative analysis of promotions based on the same reward amount. In Section 4, we focus on the immediate price reduction promotion vs. the delayed mail-in rebates promotion and analyze three cases: Endogenous price and reward amount, segmented markets, and multi-criteria retailer objective. In Section 5, we show how to estimate the needed parameters from historical transactional data. We close with our findings and directions for future research in Section 7.

2. Product-specific promotions

The retailer offers the product for a regular price of p per unit. As the duration for which the product is sold at price p increases, the number of price-sensitive consumers interested in buying the product increases, which increases the potential revenue from a promotion. When the potential revenue from a promotion becomes sufficiently large, the retailer offers a monetary promotion and then quickly returns to the regular price (Erdem, Imai, & Keane, 2003; Conlisk, Gerstner, & Sobel, 1984). The promotion can be of any of the types summarized in Table 1. The uncertainty that the consumers will receive the reward, if they purchased with the intention of getting it, and the effort required increases as we move from left to right in the table (Ailawadi, Gedenk, Langer, Ma, & Neslin, 2014).

2.1. Price reductions

In this promotion the retailer gives consumers some percentage reduction from the regular price or some $ amount off (Gupta & Cooper, 1992). Price reductions are commonly used by retailers. We are all familiar with 20%, 25%, 30%, etc. discounts on products. The discount results in an immediate reduction in the revenue of the retailer and the discount is given to all consumers. Evidence suggests that consumers discount the price discounts (Gupta & Cooper, 1992). The discounting of discounts depends on the discount level, the image of the store, and the product (brand name vs. store brand). Consumers here enjoy immediate savings without any additional effort besides purchase.

2.2. Buy-one-get-one (BOGO) discounts

In this promotion, the retailer's offer takes the form "buy X units at the regular price, get Y units at d% off". Examples are "Buy one at regular price get the second for $\frac{1}{2}$ off", "Buy one get one free", and "buy two at regular price get the third free". The consumer must purchase the first X units at the regular price to get the discount on the incremental units. This is different from when retailers, e.g. grocery stores, offer "Buy one get one free" (BOGOF) but allow consumers to buy a single unit at half price.

Table 1
Summary comparison of different promotions.

Characteristic	Promotion				
	Price reduction	Buy-one-get-one (BOGO	Coupons	Free gift cards (FGC)	Cash mail-in rebates
Sponsor	Retailer	Retailer	Retailer or manufacturer	Retailer or manufacturer*	Retailer or manufacturer
Time of savings	At purchase	At purchase	At purchase	Delayed	Delayed
Slippage	0%	0%	0%	Greater than 0%	Greater than 0%
Time of revenue reduction	Immediate	Immediate	Immediate	Delayed	Delayed
Reduction in revenue	Deterministic	Quantity dependent	Redemption dependent	Slippage dependent	Slippage dependent
Receiving consumers	All	Quantity satisfiers	Coupon holders	Card redeemers	Rebate redeemers
Cost per $1 of consumer-perceived incentive	$1	$1	$1	<$1	<$1

* Less common.

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2.3. Coupons

Coupons are vouchers or certificates which entitle consumers to a price reduction on a specific product at the time of purchase. Coupons are usually distributed via newspapers and more recently electronically (Kang, Hahn, Fortin, Hyun, & Eom, 2006). Coupons are usually offered by manufacturers and also recently by retailers (Nies & Natter, 2010). Only those consumers who expend the effort of cutting or printing the coupons and redeeming them at purchase enjoy the immediate discount. Studies indicate that only about 20 to 30% of consumers regularly go through the steps required to obtain the savings, i.e. clip, save, and then use coupons when they go shopping (Lu & Moorthy, 2007).

2.4. Free Gift Card (FGC)

With this promotion, a consumer gets a free gift card of a certain $ amount for purchasing one or more units of a product. While being the least common among the product-specific promotions, this promotion is gaining popularity (Huang, Kuo, & Shih, 2017). For example, Target offers a $5.00 free gift card for the purchase of many products such as some pet foods, over-the-counter drugs, and cosmetics. FGC combines some elements of all the above promotions. Like price reduction, FGC is given to all consumers who purchase the product. The effort required for redemption is between the effort-extensive cash mail-in rebates and the effortlessly obtained price reduction. Like rebates, FGC may suffer from slippage, i.e. non-redemption. FGC is however different from all other promotions in the sense that it does not entitle consumers to cash savings but rather to credit with the retailer. Since the cost of goods sold per $1 is less than $1 for the retailer, the cost per $1 of gift card redemption is less than $1. Also, with FGC consumers may spend more than the value of the card if they redeem the card.

One aspect of gift cards which affects their performance is the type of purchases consumers make with the gift cards. When consumers use gift cards to purchase products they would have purchased with cash in the future from the retailer, future cash sales decrease, which causes the retailer's future profit to decline (Khouja et al., 2011). We refer to the reduction of future cash sales due to gift cards' redemption as *gift card cannibalization*. In the survey conducted by Accenture, 67% of respondents stated that when redeeming gift cards, they buy themselves something they would not normally buy (Accenture, 2006), which corresponds to a 33% gift card cannibalization factor.

2.5. Cash mail-in rebate

Cash mail-in rebates are a refund to the purchaser that requires him/her to collect one or more forms, complete and mail them to the issuer to receive a cash payment sometime after purchase (Jenkins & Samiee, 2015). Rebates are used more by manufacturers than retailers. While Blattberg and Neslin (1990) describe rebates as the "durable goods analog" of coupons, Lu and Moorthy (2007) point out that a consumer responding to a coupon pays a lower price whereas a consumer responding to a rebate may still pay the regular price if she does not redeem the rebate. In other words, with rebates the savings are both uncertain and delayed, i.e. a consumer may not get the rebate and if they do, it is after purchase. Slippage rate refers to the proportion of rebates which are never redeemed and estimates of the slippage rates are about 40% (Cho, McCardle, & Tang, 2009).

3. Comparative analysis of promotions

In this section, we compare the performance of the different promotions described above. In this part of the analysis, we make some simplifying assumptions for the base comparison. We remove some of the assumptions later to gain insights into their effects.

1. The regular price and reward are predetermined. Obviously, retailers may offer different prices and rewards depending on the type of promotion they use. However, we make this assumption for the purpose of providing a base comparison of promotion effectiveness and later remove this assumption in Section 4.1. Therefore, in this section, all promotions have the same setup. We normalize a full cycle to 1 period and assume a discount of ψp, where p is the regular price per unit and $0 < \psi < 1$ is the discount factor. Thus, the price reduction, the rebate value, the free gift card value, and the coupon's face value are all of value ψp per unit. For the BOGO discount, we construct the promotion to yield ψp off per unit in the purchase of two units. For example, "buy one for regular price get the second for $2\psi p$ off" results in a discount of ψp per unit.
2. The market is made up of a single consumer segment. The utility a consumer in the segment receives from t units of the product, $u(t)$, is a uniformly distributed random variable on the range $[u(t) - b, u(t) + b]$. Consumers maximize their utility and since $u(t)$ is a concave non-decreasing function of t, it is optimal for consumers to buy at most one unit if their net utility is positive unless the promotion requires purchasing larger quantity, which is the case with BOGO promotion. The above assumption about utility results in a linear demand function in price, which is commonly used in the literature (Huang, Leng, & Parlar, 2013). This assumption may not hold in two ways. The market may still be composed of one segment but it may have different demand functions such as power or logit type functions (Huang et al., 2013). Second, the market may be segmented and demand may need to be disaggregated into segments with distinct demand functions (Dickson & Ginter, 1987). We examine the implications of a segmented market in Section 4.2.

3. Consumers do not stockpile or forward buy, which implies that promotions will result in increased consumption. Marketing researchers anticipate that promotions will be followed by a post-promotion dip in sales because of stockpiling and forward-buying. Empirical evidence of such dip has been difficult to find (Neslin & Stone, 1996). However, estimates of stockpiling are in the range of 14–50 % of the increase in sales resulting from promotions (Currim & Schneider, 1991; Gupta, 1988). Since it is reasonable to assume that stockpiling will affect all promotions in the same proportion, the results of not incorporating stockpiling will not affect the comparative analysis.
4. The retailer follows the best promotion framing. Therefore, the effects of framing on the performance of a single promotion can be ignored. For example, we do not differentiate between "buy-one-get-one free" and "buy two get 50% off" while noting the two may result in different consumer responses (Liu & Chou, 2015).
5. The retailer's objective is to maximize profit. Retailer may include other objectives such as sales, market share, and consumer surplus. We remove this assumption in Section 4.3.
6. The cost per unit of the product to the retailer is fixed. This assumption ignores the manufacturer's response to the retailer's promotion (Wierenga & Soethoudt, 2010). For example, the manufacturer may offer incentives to the retailer by lowering the unit cost for larger orders, i.e., the manufacturer may offer quantity discounts.
7. The retailer sells the product for the regular price for a duration of $1 - \tau$ followed by a discount for a short duration of τ, where $\tau \ll 1$. For the discount to be effective, τ is usually very small in the range of few days (Weathers, Swain, & Makienko, 2015) and then the retailer returns to the regular price. Gradual price increases to return to the regular price, which may result in larger profit (Tsiros & Hardesty, 2010) are not considered. We examine the effect of changes in τ in Section 4.2.
8. The promotion is offered directly from the retailer to consumers and the retailer does not allow stock-outs. We do not consider promotions offered through e-commerce intermediaries such as Groupon (Zhao, Wang, & Gan, 2016) or promotions that leave some consumer demand unsatisfied (Diels, Wiebach, & Hildebrandt, 2013).

We define the following notations

$i = d, n, m, g$, and r a subscript denoting price reduction, coupon, cash mail-in rebate, free gift card, and BOGO discount promotions, respectively,
$\alpha_i =$ the proportion of consumers who incorporate promotion i in computing the net product price,
$\delta_i =$ the discount rate applied to promotion i's reward by consumers who incorporate promotion i in computing the net product price,
$\rho_i =$ the proportion of consumers out of those incorporating promotion i in computing the net price who redeem and receive promotion i's reward,
$\gamma_i = \alpha_i \rho_i$ the proportion of all consumers who buy the product and receive promotion i's reward (for FGC and cash mail-in rebates),
$M =$ the profit per \$1 of sales,
$F =$ the fraction per \$1 of redeemed gift card used to purchase products which would not have been purchased by consumers from the retailer with cash in the future,
$1 - F=$ the gift card cannibalization factor,
$\eta =$ the additional spending per \$1 of gift card redemption,
$u(t) =$ the utility a consumer receives from t units of the product, a concave non-decreasing function of t,
$\phi =$ the proportion of consumers who wait for the promotion to purchase the product if their reservation prices are not met at the regular price, and
$x_i =$ the sales tax rate.

The parameters, α, δ, and ρ are used to capture different aspects of consumer behavior in response to delayed incentives. The parameter α reflects individuals' self-awareness and the degree to which they are time-consistent. Time-consistent individuals do not have self-control problems (Jain, 2012). Some of them know that they will not redeem the incentive and, hence, will not consider it. The rest of the time-consistent individuals know they will redeem the incentive (and they do), and use its value in computing the net price of the product. Time-inconsistent individuals incorporate the incentive into computing a net price for the product but may or may not redeem the incentive. Time-consistent consumers who know they will redeem the rebate and time-inconsistent consumers are represented by α. The parameter δ is used to express the present-bias characteristics of consumers, i.e. captures the salience of the present over the future (O'Donoghue & Rabin, 1999). Because delayed incentives are a future reward, they are discounted compared to immediate incentives. Finally, because of the time-inconsistency and self-control problems, some time-inconsistent consumers do not redeem the reward resulting in a redemption rate of $\rho \le \alpha$.

There are many marketing performance measures which can be used to compare the effectiveness of promotions. Gronholdt and Martensen (2006) list over 35 measures which are divided into four categories; financial, market, mental consumer, and behavioral consumer. Some of these measures are highly correlated. For example, one would expect market share and sales to be correlated as well as profits and earnings per share to be correlated. We select a subset of the most important measures. In the financial area, profit is considered to be one of the most critical measures (Clark, 1999) and is thus selected. In the marketing area, we use sales, which affects market share, and is considered among the top 10 most important measures (Ambler & Kokkinaki, 1997). Under our assumption of no stockouts, sales and demand can be used interchangeably. For effectiveness from a consumer satisfaction viewpoint, we use consumer surplus and price. While other measures such as consumer loyalty and perceived differentiation are important, they are quite difficult to measure. We also add a measure which is important from a societal point of view, which is the taxes collected. This measure is important because sales tax is an important source of funding for many state and local governments. Based on this discussion, we define the following promotions performance measures for $i = d, n, m, g$, and r:

1. Demand performance D_i: The total demand per discount cycle under a ψp reward of promotion type i.
2. Profit performance π_i: The retailer's total profit per discount cycle under a ψp reward of promotion type i.
3. Price performance $\bar{p}_i$: The average product price paid by all consumers under a ψp reward of promotion i.
4. Consumer surplus performance S_i: The total consumer surplus per discount cycle under a ψp reward of promotion type i. For a consumer, surplus is the difference between the amount she is willing to pay for a product and the amount she actually pays (Tewari, Youll, & Maes, 2003). Total consumer surplus is the aggregation of the surplus over all consumers who purchase the product.
5. Tax performance X_i: The total tax collected per discount cycle under a ψp reward of promotion type i.

Since we are interested in the relative performance of promotions, we use the simplest promotion, price reduction, as a benchmark and measure the effectiveness of other promotions relative to it which yields the following set of promotion effectiveness measures for $i = n, m, g$, and r:

1. Demand effectiveness $eD_i = \frac{D_i}{D_d}$: The ratio of the demand per discount cycle under a ψp reward of promotion i, $i = n, m, g$, and r, to the demand per discount cycle under a ψp reward of a price reduction promotion.
2. Profit effectiveness $e\pi_i = \frac{\pi_i}{\pi_d}$: The ratio of the retailer's profit per discount cycle under a ψp reward of promotion i, $i = n, m, g$, and r, to the profit per discount cycle under a ψp reward of a price reduction promotion.
3. Price effectiveness $e\bar{p}_i = \frac{\bar{p}_i}{\bar{p}_d}$: The ratio of the average price paid by all consumers under a ψp reward of promotion i, $i = n, m, g$, and r, to the average price paid by all consumers under a ψp reward of a price reduction promotion.
4. Consumer surplus effectiveness $eS_i = \frac{S_i}{S_d}$: The ratio of the total consumer surplus per discount cycle for all consumers under a ψp reward of promotion i, $i = n, m, g$, and r, to the total consumer surplus of all consumers per discount cycle under a ψp reward of a price reduction promotion.
5. Tax effectiveness $eX_i = \frac{X_i}{X_d}$: The ratio of taxes collected per discount cycle under a ψp reward of promotion i, $i = n, m\ g$, and r, to the taxes collected per discount cycle under a ψp reward of a price reduction promotion.

The parameter ranges for each of the promotions are shown in Table 2. These ranges are identified based on the literature. Since price reductions are given to all consumers buying the product during the promotional interval τ without requiring any effort on their part, all consumers incorporate the price reduction in computing the net price. Also, since the BOGO discount is offered to all consumers without additional effort on their part, $\alpha_r = 1$. For other promotions, some consumers decide that the reward is not worth the effort and ignore it altogether.

As discussed earlier, evidence suggests that consumers discount the reward of promotions, including price reductions. However, since we are comparing promotions and because cash has the largest appeal to consumers, we assume that it is not discounted by consumers. The same applies to coupons and BOGO discounts since they are cash savings obtained at the time of purchase. Cash mail-in rebates and free gift cards may or may not be redeemed depending on consumers' actions after-purchase and are thus discounted by consumers. There are varying estimates of redemption rates of gift cards/vouchers and rebates in the literature. Rebates are well known to have low redemption rates, i.e. high slippage. Gift cards have higher redemption rates than rebates but even gift cards with no expiration dates still suffer from slippage (Shu & Gneezy, 2010).

Suppose that the utility a consumer receives from t units of the product, $u(t)$, is a uniformly distributed random variable on the range $[u(t) - b, u(t) + b]$. Consumers maximize their utility and since $u(t)$ is a concave non-decreasing function of t, it is optimal for consumers to buy at most one unit if their net utility is positive unless the promotion requires purchasing larger quantity, which is the case with BOGO discount. The above assumption about utility results in the commonly used linear demand function in price (Huang et al., 2013).

We assume a consumer arrival rate of λ per unit time. In this section, for comparison purposes, we normalize the arrival rate to $\lambda = 1$ and assume the per unit cost of the product for the retailer is zero. Let $u(t = 1) = u_1$, i.e. the utility from one unit

Table 2
Parameter ranges of different promotions.

Parameter	Promotion: Price reduction (d)	Coupons (n)	Cash mail-in rebates (m)	Free gift cards (g)	BOGO (r)
Proportion of consumers who incorporate i, α_i	$a_d = 1$	$\alpha_n < 1$	$\alpha_m < 1$	$\alpha_g < 1$	$\alpha_r = 1$
Discount rate applied to promotion i's reward, δ_i	$\delta_d = 0$	$\delta_n > 0$	$\delta_m =>$	$\delta_g > 0$	$\delta_r = 0$
Proportion of buyers in α_i who receive promotion i's reward, ρ_i	$\rho_d = 1$	$\rho_n = 1$	$\rho_m < 1$	$\rho_g < 1$	$\rho_r = 1$
Proportion of all buyers who receive promotion i's reward, $\gamma_i = \alpha_i \rho_i$	$\gamma_d = 1$	$\gamma_n < 1$	$\gamma_m < 1$	$\gamma_g < 1$	$\gamma_r < 1$

of the product, a uniformly distributed random variable on $[u_1 - b, u_1 + b]$. Without promotions, the demand and profit of the retailer per period are $D = \int_p^{u_1+b} \frac{1}{2b}\, du = \frac{u_1+b-p}{2b}$ and $\pi = p\frac{u_1+b-p}{2b}$. Since π is concave, the optimal price, demand, profit, consumer surplus, and taxes collected are:

$$p^* = \frac{u_1 + b}{2}, \tag{1}$$

$$D^* = \frac{u_1 + b}{4b}, \tag{2}$$

$$\pi^* = \frac{(u_1 + b)^2}{8b}, \tag{3}$$

$$S^* = \frac{(u_1 + b)^2}{16b}, \tag{4}$$

and

$$X^* = \frac{r(u_1 + b)^2}{8b}, \tag{5}$$

respectively. For simplicity, we assume that the retailer sets the price at the regular period to the optimal price given in Eq. (1). Consumers whose reservation prices are not met, i.e. consumers with reservation prices on $[u_1 - b, p^*]$, accumulate in the population of potential consumers (Conlisk et al., 1984). Also, for simplicity we assume all pent-up demand consumers wait in the market and set $\phi = 1$, which does not affect the analysis of the relative performance of promotions.

Now, suppose the retailer offers a promotion during the last τ days per period. During the non-promotion part of the period, $1 - \tau$, the profit of the retailer is

$$\pi_d = p^*(1-\tau)\int_{p^*}^{u_1+b} \frac{1}{2b}\, du = \frac{(1-\tau)(b+u_1-p)p}{2b} = \frac{(1-\tau)(u_1+b)^2}{8b}. \tag{6}$$

For a price reduction promotion of ψp^* ($\psi \times 100\%$ of price) during τ, the resulting price is $p^* - \psi p^*$. Let $\omega = b + u_1$, then, the proportion of the pent-up demand which accumulated over the regular price period of $1 - \tau$ on the range $[p^* - \psi p^*, p^*]$ is $(1-\tau)\int_{p^*-\psi p^*}^{p^*} \frac{1}{2b} du = (1-\tau)\frac{\psi\omega}{4b}$. These consumers will purchase the product during τ due to the price reduction. The realized demand from consumers arriving over the discount period of τ is $\tau\int_{p^*-\psi p^*}^{\omega} \frac{1}{2b}\, du = \frac{\tau(\omega-p^*+\psi p^*)}{2b} = \frac{\tau\omega(1+\psi)}{4b}$. The total demand of consumers arriving during $(1 - \tau)$ and buying the product for p^* is $(1-\tau)\int_{p^*}^{\omega} \frac{1}{2b}\, du = (1-\tau)\frac{\omega-\frac{\omega}{2}}{2b}$. The total demand and profit in the entire discount cycle under the price reduction promotion are

$$D_d = (1-\tau)\frac{\psi\omega}{4b} + \frac{\tau\omega(1+\psi)}{4b} + (1-\tau)\frac{\omega-\frac{\omega}{2}}{2b} = \frac{(1+\psi)\omega}{4b}, \tag{7}$$

and

$$\pi_d = \frac{(1+\psi(1-\tau-\psi))\omega^2}{8b}, \tag{8}$$

respectively. If the applicable sales tax rate is r, then the taxes, average price paid by consumers, and consumer surplus with the ψp^* price reduction are

$$x_d = r\,\frac{(1+\psi(1-\tau-\psi))\omega^2}{8b}, \tag{9}$$

$$\bar{p}_d = \frac{(1+\psi(1-\tau-\psi))\omega}{2(1+\psi)}, \tag{10}$$

and

$$S_d = \frac{(1+2\tau\psi+\psi^2)\,\omega^2}{16b}, \tag{11}$$

respectively.

For FGC, the demand, profit, average price, consumer surplus, and the taxes collected have additional consideration in their calculations. This is due to the *gift card cannibalization*, $F_1 = (1 - F)$, which we defined as the reduction in future cash sales because of gift cards redemption (Khouja et al., 2011). The cost per \$1 of gift card redemption to the retailer is \$$(1 - M)$.

Furthermore, a gift card redeemer uses on average $100 \times (1-F)\%$ of her gift card to pay for purchases she would have made with cash in the future at the retailer, resulting in a decline in the retailer's future profit of \$ $M(1-F)$ per \$1 of gift card redemption. Therefore, the retailer's net cost per \$1 of gift card redemption is $\theta = 1 - M + M(1-F) = \$(1-FM)$. An $F = 1$ results in $\theta = 1 - M$ which is the cost of goods sold fraction. An $F = 0$ implies that for each \$1 of redeemed gift cards the retailer incurs a cost \$1. In other words, if a consumer spends the gift card to purchase a product she/he regularly purchases from the retailer, then the cost to the retailer is the same as a price reduction of the card's face value. Based on the above, the product demand is

$$\begin{aligned} D_g =& \frac{\lambda\tau\alpha_g\left(\omega-\left(p-\psi p\left(1-\delta_g\right)\right)\right)}{2b}+\frac{\alpha_g\lambda(1-\tau)\left(p-\left(p-\psi p\left(1-\delta_g\right)\right)\right)}{2b} \\ &+\frac{\lambda(\omega-p)\left((1-\tau)+\tau\left(1-\alpha_g\right)\right)}{2b}=\frac{\omega\left(1+\psi\alpha_g\left(1-(1+\tau)\delta_g\right)\right)}{4b}. \end{aligned} \tag{12}$$

The total profit includes the profit from product sale as well as the cost and benefit from gift card redemption

$$\begin{aligned} \pi_g =& \left[\frac{\lambda\tau\alpha_g\left(\omega-\left(p-\psi p(1-\delta_g)\right)\right)}{2b}+\frac{\alpha_g\lambda(1-\tau)\left(p-\left(p-\psi p(1-\delta_g)\right)\right)}{2b}\right](p-\psi p) \\ &+\frac{\lambda(\omega-p)\left((1-\tau)+\tau(1-\alpha_g)\right)}{2b}p+\frac{\left(\omega\alpha_g\left(\tau+\psi-(1+\tau)\psi\delta_g\right)\right)\left(\eta\alpha_g\rho_g MF\psi p-(1+\eta)\alpha_g\rho_g\psi p(1-M)\right)}{4b} \\ =& \frac{\omega^2\left(1-\tau\alpha_g+\alpha_g\left(\tau+\psi-(1+\tau)\psi\delta_g\right)\left(1-(1-M+(1-M-FM)\eta)\psi\alpha_g\rho_g\right)\right)}{8b}. \end{aligned} \tag{13}$$

The consumer surplus, and the average price paid by consumers are

$$S_g = \frac{\omega^2\left(1-\psi\alpha_g\left(\psi\left(1-\delta_g\right)^2-2\left(\tau+\psi-\psi\delta_g\right)\rho_g\right)\right)}{16b}, \tag{14}$$

and

$$\bar{p}_g = \frac{\omega\left(1-\psi\alpha_g\left((1+\tau)\delta_g+\alpha_g\left(\tau+\psi-(1+\tau)\psi\delta_g-1\right)\rho_g\right)\right)}{2+2\psi\alpha_g\left(1-(1+\tau)\delta_g\right)} \tag{15}$$

respectively, and taxes collected are $r\,\pi_g$.

Another type of promotion which requires further examination is the BOGO discount. Consumer i receives a utility of $u_{1,i} \sim [0,1]$ from the first unit and $\beta_i u_{1,i}$ from the second unit. We assume that consumers are homogenous in the sense that they have the same β. Assuming β_i to be random variable will result in complicated analysis involving the convolution of random variables and will detract from our focus of comparing promotions. If consumers have the same β, then there are four consumer groups:

1. Consumers with reservation prices satisfying $u - p < 0$ who arrive during $1-\tau$. These consumers become pent-up demand consumers whose utility is uniformly distributed on $[0, p^*]$ and will maximize their utility by buying two units if $u + \beta u \geq 2(p^* - \psi p^*)$, which gives $u \geq \frac{2(p^*-p^*\psi)}{\beta+1}$. Therefore, the demand from this group is $2(1-\tau)\int_{\frac{2(p^*-\psi p^*)}{1+\beta}}^{p^*}\frac{1}{2b}\,du = 2(1-\tau)\int_{\frac{\omega-\psi\omega}{1+\beta}}^{\omega/2}\frac{1}{2b}\,du = \frac{(1-\tau)(\beta+2\psi-1)\omega}{2(1+\beta)b}$. Therefore, for β to create any additional sales from this group, it must satisfy

   $$\beta > 1 - 2\psi \equiv \hat{\beta}$$

2. Consumers with reservation prices satisfying $u - p < 0$ who arrive during τ. These consumers behave the same way as the first group and their demand is $2\tau\int_{\frac{2(p-\psi p)}{1+\beta}}^{p}\frac{1}{2b}\,du = \frac{\tau(\beta+2\psi-1)\omega}{2b(1+\beta)}$.
3. Consumers with reservation prices satisfying $u - p > 0$ who arrive during τ. By condition (1), these consumers have $u(1+\beta) - 2(p-\psi p) > u - p$ and will buy two units. Thus, the demand from this group is $2\tau\int_{\omega/2}^{\omega}\frac{1}{2b}\,du = \frac{\tau\omega}{2b}$
4. Consumers who arrive during $1-\tau$ with $u \geq p$ and buy one unit. Their demand is $\frac{(1-\tau)\omega}{4b}$.

Thus, the total demand is the sum of the demand from the above four groups which yields

$$D_r = \frac{(\tau+\beta(3+\tau)+4\psi-1)\omega}{4(1+\beta)b}. \tag{16}$$

The profit, average price paid by consumers, and their surplus are

$$\pi_r = \frac{\left(3\beta+\tau+\beta\tau-2(3-\beta-\tau-\beta\tau)\psi+4\psi^2-1\right)\omega^2}{8(1+\beta)b}, \tag{17}$$

$$\bar{p}_r = \frac{(3\beta + \tau + \beta\tau + 2(3 - \beta - \tau - \beta\tau)\psi - 4\psi^2 - 1)\,\omega}{2(\tau + \beta(3 + \tau) + 4\psi - 1)}, \tag{18}$$

and

$$S_r = \frac{(3 - 3\tau - \beta(3 - 2\beta - 3\tau - 6\beta\tau) - 8\psi + 8(\beta + \tau + \beta\tau)\psi + 8\psi^2)\,\omega^2}{16(1 + \beta)b}, \tag{19}$$

respectively. The amount of taxes collected is ($r \times \pi_r$). Similar analysis for coupons and cash mail-in rebates yield the demands, profits, average prices, consumer surplus, and taxes for each of the promotions shown in Table 3.

Since we are using price reduction as the benchmark, we first examine its performance relative to no-promotions. A price reduction of ψp results in

$$\frac{D_d - D^*}{D^*} = 100\psi\% \quad \textit{change (increase) in demand} \tag{20}$$

$$\frac{\pi_d - \pi^*}{\pi^*} = 100\psi(1 - \tau - \psi)\% \quad \textit{change (increase) in profit} \tag{21}$$

$$\frac{\bar{p}_d - \bar{p}}{\bar{p}} = 100\frac{2\psi(\tau + \psi)}{1 + \psi}\% \quad \textit{change (decrease) in average price} \tag{22}$$

$$\frac{S_d - S^*}{S^*} = 100\psi(2\tau + \psi)\% \quad \textit{change (increase) in consumer surplus} \tag{23}$$

$$\frac{X_d - X^*}{X^*} = 100\psi(1 - \tau - \psi)\% \quad \textit{change (increase) in taxes collected} \tag{24}$$

Since we are interested in the relative effectiveness of promotions, we divide each effectiveness measures of coupon, cash mail-in rebate, FGC, and BOGO discount by the corresponding measures for price reduction promotion in Eqs. (7)–(10) to get the expressions in Table 4. For simplicity, since τ is very small, we set $\tau = 0$ in this section. In Proposition 1, we show that if consumer utility from the second unit in the BOGO discount is large (close to the utility from the first unit), then the BOGO discount has the best price effectiveness (lowest average price). Otherwise, price reduction is best in terms of price effectiveness. All proofs are contained in the Appendix.

Proposition 1. *For a promotion of ψp, if $\beta > \frac{2-4\psi+\psi\alpha_i(1-\delta_i)(2-4\psi-(1-4\psi)\alpha_i\rho_i)}{2+\psi\alpha_i(1-\delta_i)(2-3\alpha_i\rho_i)}$, $i = g, m$, and $\beta > \frac{4(1-\psi)}{2-\psi\alpha_n} - 1$, then BOGO discount has the lowest average price. Otherwise, price reduction has the lowest average price of all promotions.*

Table 3
Demand, profit, average price, consumer surplus, and taxes for coupons and rebates.

	Coupons	Cash mail-in rebates
Demand	$\frac{\omega(1+\psi\alpha_n)}{4b}$	$\frac{\omega(1+\psi\alpha_m(1-\delta_m))}{4b}$
Profit	$\frac{\omega^2(1+\psi(1-\tau-\psi)\alpha_n)}{8b}$	$\frac{\omega^2(1+\psi\alpha_m(1-\delta_m-\alpha_m(\tau+\psi-\psi\delta_m)\rho_m))}{8b}$
Average price	$\frac{\omega+\psi(1-\tau-\psi)\omega\alpha_n}{2+2\psi\alpha_n}$	$\frac{\omega(1+\psi\alpha_m(1-\delta_m-\alpha_m(\tau+\psi-\psi\delta_m)\rho_m))}{2+2\psi\alpha_m(1-\delta_m)}$
Consumer surplus	$\frac{\omega^2(1+\psi(2\tau+\psi)\alpha_n)}{16b}$	$\frac{\omega^2\left(1-\psi\alpha_m\left(\psi(1-\delta_m)^2-2(\tau+\psi-\psi\delta_m)\rho_m\right)\right)}{16b}$
Taxes	$r\ \frac{\omega^2(1+\psi(1-\tau-\psi)\alpha_n)}{8b}$	$\frac{x\omega^2(1+\psi\alpha_m(1-\delta_m))}{8b}$

Table 4
Ratio of measures under different promotions to price reduction measures.

	Coupons	Cash mail-in rebates
Demand	$\frac{1+\psi\alpha_n}{1+\psi}$	$\frac{1+\alpha_m\psi(1-\delta_m)}{1+\psi}$
Profit	$\frac{1+(1-\psi)\psi\alpha_n}{1+(1-\psi)\psi}$	$\frac{1+\psi\alpha_m(1-\delta_m)(1-\psi\alpha_m\rho_m)}{1+(1-\psi)\psi}$
Price	$\frac{(1+\psi)(1+(1-\psi)\psi\alpha_n)}{(1-\psi)(1+2\psi)(1+\psi\alpha_n)}$	$\frac{(1+\psi)(1+\psi\alpha_m(1-\delta_m)(1-\psi\alpha_m\rho_m))}{(1-\psi)(1+2\psi)(1+\psi\alpha_m(1-\delta_m))}$
Consumer surplus	$\frac{1+\psi^2\alpha_n}{1+\psi^2}$	$\frac{1-\psi^2\alpha_m(1-\delta_m)(1-\delta_m-2\rho_m)}{1+\psi^2}$
Taxes	$\frac{1+(1-\psi)\psi\alpha_n}{1+(1-\psi)\psi}$	$\frac{1+\psi\alpha_m(1-\delta_m)}{1+\psi-\psi^2}$
	Free gift card	**BOGO discount**
Demand	$\frac{1+\alpha_g\psi(1-\delta_g)}{1+\psi}$	$\frac{3\beta+4\psi-1}{(1+\beta)(1+\psi)}$
Profit	$\frac{1+\psi\alpha_g(1-\delta_g)(1+(M-(1-M-FM)\eta-1)\psi\alpha_g\rho_g)}{1+(1-\psi)\psi}$	$\frac{3\beta+2(3-\beta)\psi-4\psi^2-1}{(1+\beta)(1+(1-\psi)\psi)}$
Price	$\frac{(1+\psi)(1+\psi\alpha_g(1-\delta_g)(1-\psi\alpha_g\rho_g))}{(1-\psi)(1+2\psi)(1+\psi\alpha_g(1-\delta_g))}$	$\frac{(1+\psi)\left(3\beta+2(3-\beta)\psi-4\psi^2-1\right)}{(1-\psi)(1+2\psi)(3\beta+4\psi-1)}$
Consumer surplus	$\frac{1-\psi^2\alpha_g(1-\delta_g)(1-\delta_g-2\rho_g)}{1+\psi^2}$	$\frac{3+2\beta^2-8(1-\psi)\psi-\beta(3-8\psi)}{(1+\beta)\left(1+\psi^2\right)}$
Taxes	$\frac{(1+\psi\alpha_g(1-\delta_g))(1+(1-F)M(1+\eta)\psi\alpha_g\rho_g)}{1+(1-\psi)\psi}$	$\frac{3\beta+2(3-\beta)\psi-4\psi^2-1}{(1+\beta)(1+(1-\psi)\psi)}$

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Proposition 1 is illustrated in Fig. 1. We do not include coupons in this figure and subsequent figures since coupons performance is always dominated by price reduction performance. While FGC and cash in-mail rebates can never have a lower average price than price reduction, BOGO discounts can when the utility from the second unit (β) is 0.715 of the utility from the first unit or larger. With FGC and cash mail-in rebates, the average price decreases with the redemption rate.

In Proposition 2, we show that if consumer utility from the second unit in the BOGO discount is large and the number of forward-looking consumers who stockpile is small, then the BOGO discount has the best demand effectiveness (largest demand). Otherwise, price reduction has the best demand effectiveness.

Proposition 2.

1. *If* $\beta > 3 - \frac{4}{2-\psi}$ *then BOGO discount has the highest demand effectiveness of all promotions.*
2. *If* $\beta < 3 - \frac{4}{2-\psi}$ *then price reduction has the highest demand effectiveness of all promotions.*
3. *If* $\beta = 3 - \frac{4}{2-\psi}$ *then BOGO discount and price reduction have the same demand effectiveness which is higher than all other promotions.*

Proposition 2 is illustrated in Fig. 2. Again FGC and cash in-mail rebates can never stimulate more demand than a price reduction, and BOGO discounts can result in substantially higher sales when the utility from the second unit (β) is larger than 0.715 of the utility from the first unit and no stockpiling. With FGC and cash mail-in rebates, the demand and redemption rates are independent of each other.

In Proposition 3, we show that from a consumer surplus perspective, cash mail-in rebates, FGC, and coupons can't outperform price reduction, and when the utility from the second unit is large, those three promotions can't outperform BOGO discounts either.

Proposition 3 is illustrated in Fig. 3. Both FGC and cash mail-in rebates have poor performance in terms of increasing consumer surplus while BOGO again can perform well when β is large. If all consumers redeem FGC and cash mail-in rebates,

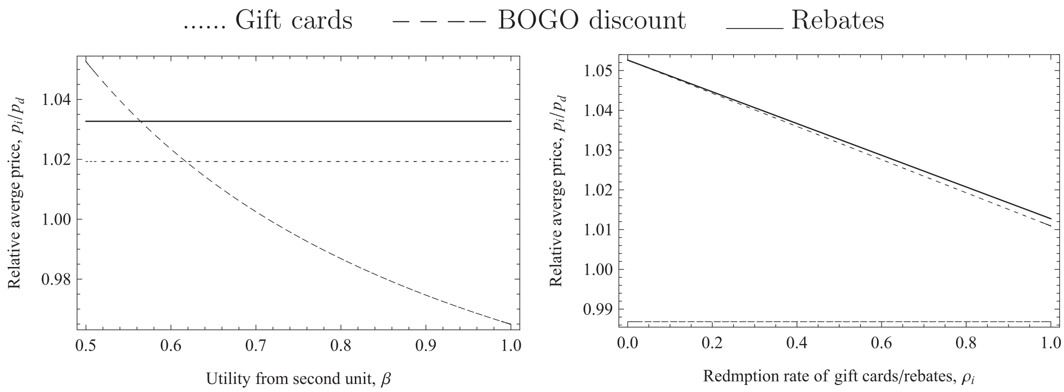

Fig. 1. Price performance of different promotions.
$u_1 = 100, b = 40, p_1 = 110, r = 0007, \eta = 0.3, F = 05, M = 0.50, \alpha_m = \alpha_g = 0.9, \delta_m = \delta_g = 0.1, \rho_m = 0.5, \rho_g = 0.8.$

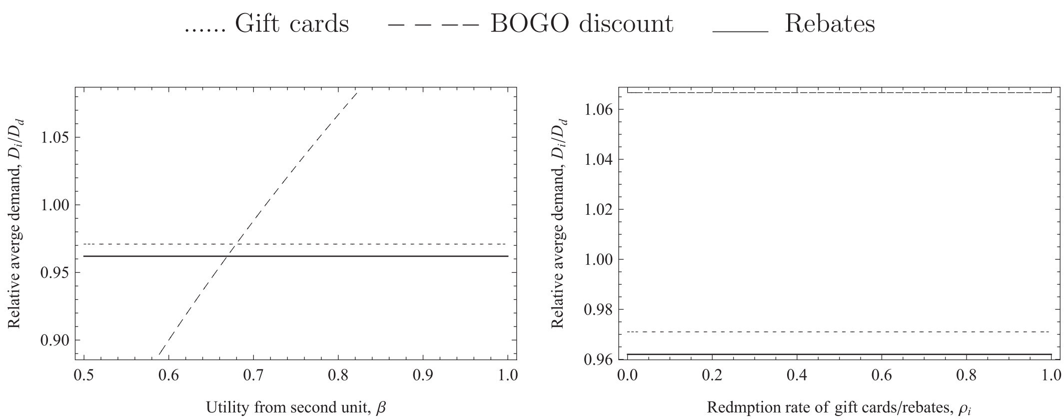

Fig. 2. Demand performance of different promotions.
$u_1 = 100, b = 40, p_1 = 110, r = 0.07, \eta = 0.3, F = 0.5, M = 0.50, \alpha_m = \alpha_g = 0.9, \delta_m = \delta_g = 0.1, \beta = 0.8, \rho_m = 0.5, \rho_g = 0.8.$

then, these two promotions can at best match the price reduction consumer surplus. The reason that FGC and rebates can't provide higher consumer surplus is two fold. First, not all consumers take them into account when they make a purchase decision because they mistrust them and view them with resentment, especially for the case of rebates. Second, some consumers may not redeem them.

Proposition 3. *If* $\beta > \frac{1}{4}\left(4-\psi\left(8-\psi-\sqrt{16-(16-\psi)\psi}\right)\right)$ *then BOGO discount has the highest consumer surplus effectiveness and if* $\beta < \frac{1}{4}\left(4-\psi\left(8-\psi-\sqrt{16-(16-\psi)\psi}\right)\right)$ *then price reduction has the highest consumer surplus effectiveness.*

In Proposition 4, we show that unlike price and demand effectiveness, cash mail-in rebates and FGC can have the highest profit effectiveness of all promotions.

Proposition 4.

1. *If* $\beta < 3 - \frac{4}{2-\psi}$ *then price reduction has higher profit effectiveness than BOGO discount and*
   - *If* $0 \le \rho_m < \frac{\psi+\alpha_m(1-\delta_m)-1}{\psi\alpha_m^2(1-\delta_m)}$ *and/or* $0 \le \rho_g < \frac{\psi+\alpha_g(1-\delta_g)-1}{(1-R+(1-R-FR)\eta)\psi\alpha_g^2(1-\delta_g)}$, *then gift cards and/or cash mail-in rebates have highest profit effectiveness,*
   - *If* $\rho_m > \frac{\psi+\alpha_m(1-\delta_m)-1}{\psi\alpha_m^2(1-\delta_m)}$ *and* $\rho_g > \frac{\psi+\alpha_g(1-\delta_g)-1}{(1-R+(1-R-FR)\eta)\psi\alpha_g^2(1-\delta_g)}$, *then BOGO discount has the highest profit effectiveness.*
2. *If* $\beta > 3 - \frac{4}{2-\psi}$ *then BOGO discount has higher profit effectiveness than price reduction and*
   - *If* $0 \le \rho_m < \bar{\rho}_m \equiv \frac{(1+\beta)(1+\psi\alpha_m(1-\delta_m)(1-\psi\alpha_m\rho_m))}{3\beta+2(3-\beta)\psi-1-4\psi^2}$ *and/or* $0 \le \rho_g < \frac{2(1-\psi)(1-\beta-2\psi)+(1+\beta)\psi\alpha_g(1-\delta_g)}{(1+\beta)(1-R+(1-R-FR)\eta)\psi^2\alpha_g^2(1-\delta_g)}$, *then gift cards and/or cash mail-in rebates have highest profit effectiveness,*
   - *If* $\rho_m > \frac{(1+\beta)(1+\psi\alpha_m(1-\delta_m)(1-\psi\alpha_m\rho_m))}{3\beta+2(3-\beta)\psi-1-4\psi^2}$ *and* $\rho_g > \frac{2(1-\psi)(1-\beta-2\psi)+(1+\beta)\psi\alpha_g(1-\delta_g)}{(1+\beta)(1-R+(1-R-FR)\eta)\psi^2\alpha_g^2(1-\delta_g)}$, *then BOGO discount has the highest profit effectiveness.*

Proposition 4 is illustrated in Fig. 4. FGC can have better profit performance than price reduction even with a redemption rate ρ_m up to 0.80. Furthermore, the decline in profit performance as the redemption rate increases is much steeper for cash mail-in rebates than FGC. Also, BOGO has higher profit than price reduction if the utility from the second unit (β) is 0.715 of the utility from the first unit or larger. The strong performance of BOGO and FGC is supported by their rising popularity among many retailers.

In Proposition 5 we show that FGC and cash mail-in rebates may outperform price reduction as well as BOGO discount for taxes, which can't happen for price, demand, and consumer surplus effectiveness.

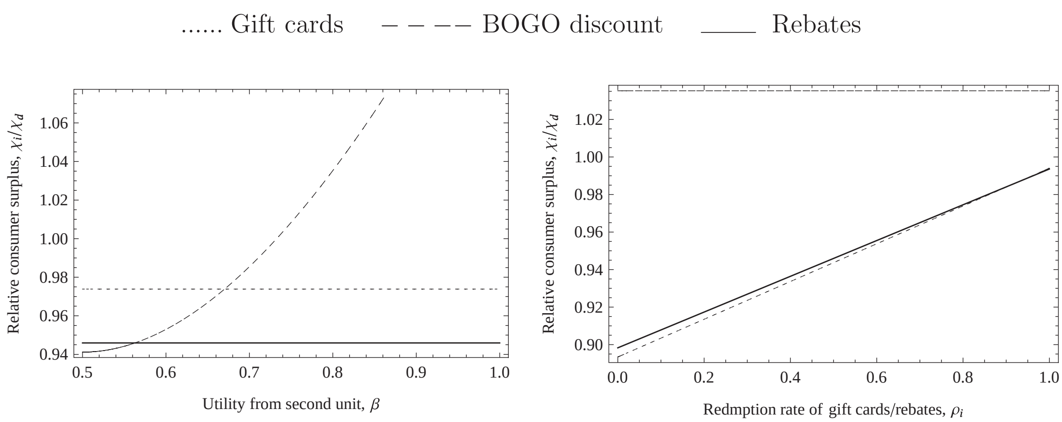

Fig. 3. Consumer surplus performance of different promotions.
$u_1=100, b=40, p_1=110, r=0.07, \eta=0.3, F=0.5, M=0.50, \alpha_m=\alpha_g=0.9, \delta_m=\delta_g=0.1, \beta=0.8, \rho_m=0.5, \rho_g=0.8.$

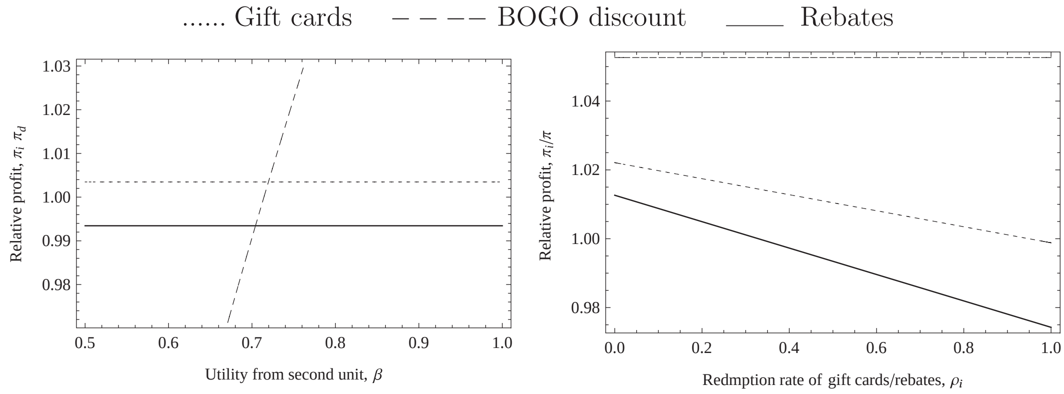

Fig. 4. Profit performance of different promotions.
$u_1 = 100$, $b = 40$, $p_1 = 110$, $r = 0.07$, $\eta = 0.3$, $F = 0.5$, $M = 0.50$, $\alpha_m = \alpha_g = 0.9$, $\delta_m = \delta_g = 0.1$, $\beta = 0.8$.

Proposition 5.

1. *If* $\beta < 3 - \frac{4}{2-\psi}$, *then price reduction has higher tax effectiveness than BOGO discount and*
   (a) *If* $\alpha_m > \frac{1-\psi}{1-\delta_m}$ *and/or* $\rho_g > \frac{1-\psi-\alpha_g(1-\delta_g)}{(1-F)R(1+\eta)\alpha_g(1+\psi\alpha_g(1-\delta_g))}$ *then gift cards and/or cash mail-in rebates have highest tax effectiveness.*
   (b) *If* $\alpha_m < \frac{1-\psi}{1-\delta_m}$ *and* $\rho_g < \frac{1-\psi-\alpha_g(1-\delta_g)}{(1-F)R(1+\eta)\alpha_g(1+\psi\alpha_g(1-\delta_g))}$ *then price reduction has the highest tax effectiveness.*
2. *If* $\beta > 3 - \frac{4}{2-\psi}$ *, then*
   (a) *If* $\alpha_m > \frac{2(1-\psi)(\beta+2\psi-1)}{(1+\beta)\psi(1-\delta_m)}$ *and/or* $\rho_g > \frac{2(1-\psi)(2\psi-1+\beta)-(1+\beta)\psi\alpha_g(1-\delta_g)}{(1-F)R(1+\beta)(1+\eta)\psi\alpha_g(1+\psi\alpha_g(1-\delta_g))}$ *then gift cards and/or cash mail-in rebates have highest tax effectiveness.*
   (b) *If* $\alpha_m < \frac{2(1-\psi)(\beta+2\psi-1)}{(1+\beta)\psi(1-\delta_m)}$ *and/or* $\rho_g < \frac{2(1-\psi)(2\psi-1+\beta)-(1+\beta)\psi\alpha_g(1-\delta_g)}{(1-F)R(1+\beta)(1+\eta)\psi\alpha_g(1+\psi\alpha_g(1-\delta_g))}$ *then BOGO discount has the highest tax effectiveness.*

Proposition 5 is illustrated in Fig. 5. FGC and cash mail-in rebates have the best performance unless β is very large. Furthermore, the tax performance of FGC can substantially improve with increased redemption rate whereas the performance of cash mail-in rebates is independent of it. The strong FGC tax performance is because consumers pay the full price p at purchase and the card redemption is taxed as well.

4. Extensions

In Section 3, to provide a base comparison between promotions, we made some simplifying assumptions. In this section, we remove some of the more restrictive assumptions, one at a time. We focus our analysis on comparing one immediate incentive, price reduction, and one delayed incentive, cash mail-in rebate. Since the analysis is of increasing complexity, we now assume that $\omega = 1$ and $b = \frac{1}{2}$, which implies the commonly used assumption that consumer reservation prices are uniformly distributed on the interval $[0, 1]$ (Shin, 2005). For the following corollaries, the results are obtained directly from the accompanying tables and therefore, their proofs are omitted. We begin by allowing both the regular price and the discount price, and therefore the reward, to be decision variables.

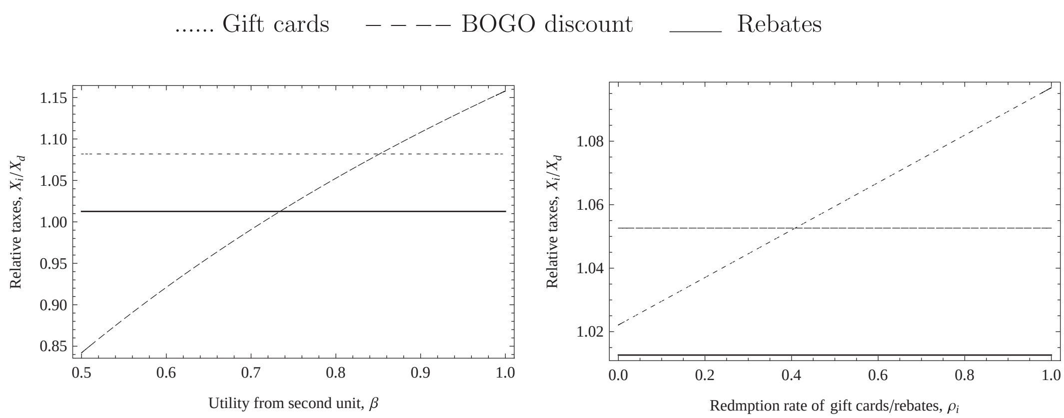

Fig. 5. Tax performance of different promotions.
$u_1 = 100$, $b = 40$, $p_1 = 110$, $r = 0.07$, $\eta = 0.3$, $F = 0.5$, $M = 0.50$, $\alpha_m = \alpha_g = 0.9$, $\delta_m = \delta_g = 0.1$, $\beta = 0.8$.

4.1. Endogenous regular and discount prices

In this extension, both the regular price p and the discount price ψp are decision variables to be determined to maximize the retailer's profit. For price reduction, the profit is $\pi_d = p\left(1 - \tau_d \psi_d - p_d\left(1 - (1 + \tau_d)\psi_d - \psi_d^2\right)\right)$. In the case of the rebate, the retailer maximizes $\pi_m = p_m\left(1 - p_m + p_m \psi_m \alpha_m (1 - \delta_m)(1 - \psi_m \rho_m)\right)$. The optimal regular prices, discount prices, demand, and profit are shown in Table 5. Comparing the results from Table 5, we can prove the Corollary 1.

Corollary 1.

1. *If $\rho_m < \alpha_m(1 - \delta_m)$, then*
   (a) *cash mail-in-rebates promotions are more profitable than price reduction promotions;*
   (b) *the regular price is higher when the retailer uses cash mail-in rebates promotions;*
   (c) *the value of the rebate is larger than the price difference in the price reduction promotion;*
   (d) *the quantity sold will be larger with rebates; and*
   (e) *the taxes collected will be larger with rebates.*
2. *If $\rho_m > \alpha_m(1 - \delta_m)$, then price reduction promotions are more profitable than cash mail-in rebates promotions.*

Corollary 1 has important implications. Considering estimates of slippage rates are about 40% (Hu, Hu, & Ye, 2017), one would expect the condition $\rho_m < \alpha_m(1 - \delta_m)$ to be satisfied in realistic settings. However, the decrease in rebate popularity among retailers may indicate otherwise. Because fewer consumers even consider rebates in their mental accounting when making a purchase decision and consumers who consider rebates apply large discount factors to them, $\rho_m < \alpha_m(1 - \delta_m)$ may not hold. For example, if only 60% of consumers consider the rebate and discount its value by 30% then a redemption rate of less than 42% is needed for rebates to be more profitable than a price reduction promotion. Another negative aspect of rebates is that when the retailer uses them, the reward will be larger than in a price reduction. Consumers who redeem the rebate will get higher savings but their savings are being subsidized by consumers who incorporated the discount in their mental accounting but failed to redeem it. Therefore, while the retailer and some consumers are better off, many other consumers are worse off.

If $\rho_m = 0.6$, then α_m must satisfy $\alpha_m < \frac{24}{5\left(17 - \delta_m(32 - 15\delta_m) + (1 - \delta_m)\sqrt{3(59 - 5\delta_m(34 - 15\delta_m))}\right)}$ for rebates to result in higher consumer surplus than price reductions. This major drawback of rebates is illustrated in Fig. 6. As the figure shows, in-spite of the value of the rebate being larger than the price reduction, for realistic values of α_m and δ_m rebates will result in lower consumer surplus than a price reduction promotion. For example, for $\rho_m = 0.6$ if $\alpha_m = 0.50$, consumers must discount the rebate reward by at least 0.377 for rebates to result in larger surplus, and this discounting of the rebate reward needs to be higher for ρ_m smaller than 0.6. Note that for small ρ_m and δ_m and large α_m, the rebate reward can be very large, which may explain the "free after rebate" offers shown in Fig. 10 commonly advertised in the past before rebates developed their negative image among consumers.

4.2. Segmented market and promotion duration

Suppose the retailer faces two consumer segments, a price-insensitive segment (Tellis, 1986), denoted by the subscript t, and a price-sensitive segment, denoted by the subscript s. The arrival rate of all consumers is $\lambda = \lambda_t + \lambda_s = 1$ and $0 < \lambda_t < 1$. Consumers in segment s, knowing that a promotion is coming, will not buy the product until the promotion. Consumers in segment λ_t will buy the product upon arrival if their reservation prices are met. We assume price insensitive consumers have reservation prices larger than one for the product. The retailer sells to all price-insensitive consumers by setting the price to $p = 1$ and then sells to price-sensitive consumers who waited by reducing the price during τ. With a price reduction promotion and a regular price of $p = 1$, the profit of the retailer is $\pi_d = (1 - \psi_d)\psi_d + \left(1 - (1 + \tau_d)\psi_d + \psi_d^2\right)\lambda_t$. Since $\frac{d\pi_d}{d\tau} = -\psi\lambda_t \le 0$, the profit is decreasing in the duration of the discount and it is optimal for the retailer to set the promotion duration to the minimum needed to give price-sensitive consumers enough time to buy the product. By using the minimum duration, denoted by τ_d, fewer price-insensitive consumers arriving during the short τ_d get the discount. This may be one of the reasons the literature suggests that for the discount to be effective, τ is usually very small in the range of few days (Weathers et al., 2015). The optimal discount price, demand, profit, and consumer surplus are obtained from the first order condition as $p_d^* = \frac{1}{2}\left(1 + \frac{\tau_d \lambda_t}{1 - \lambda_t}\right)$, $D_d^* = \frac{1}{2}\left(1 + (1 - \tau_d)\lambda_t\right)$ and $\pi_d^* = \frac{1 + (1 - \tau_d)\lambda_t(2 - (3 + \tau_d)\lambda_t)}{4(1 - \lambda_t)}$, and $\chi_d^* = \frac{(1 - (1 + \tau_d)\lambda_t)(1 - (1 - 3\tau_d)\lambda_t)}{8(1 - \lambda_t)}$ respectively. Solving $\psi_d^* = 0$, (i.e., no price reduction) yields $\lambda_t = \frac{1}{1 + \tau_d}$. This implies that if the duration of the discount needed for selling to the price-sensitive

Table 5
Measures of promotions performance with endogenous regular and discount prices.

	Price reduction	Cash mail-in rebates
Demand	$\frac{2}{3}$	$\frac{2\rho_m}{4\rho_m - \alpha_m(1 - \delta_m)}$
Profit	$\frac{1}{3}$	$\frac{\rho_m}{4\rho_m - \alpha_m(1 - \delta_m)}$
Regular price	$\frac{2}{3}$	$\frac{2\rho_m}{4\rho_m - \alpha_m(1 - \delta_m)}$
Discount price	$\frac{1}{3}$	$\frac{2\rho_m - 1}{4\rho_m - \alpha_m(1 - \delta_m)}$
Consumer surplus	$\frac{1}{9}$	$\frac{2\alpha_m(1 - \delta_m)\rho_m - 4\rho_m^2 - (1 - \alpha_m)\alpha_m(1 - \delta_m)^2}{2(4\rho_m - \alpha_m(1 - \delta_m))^2}$

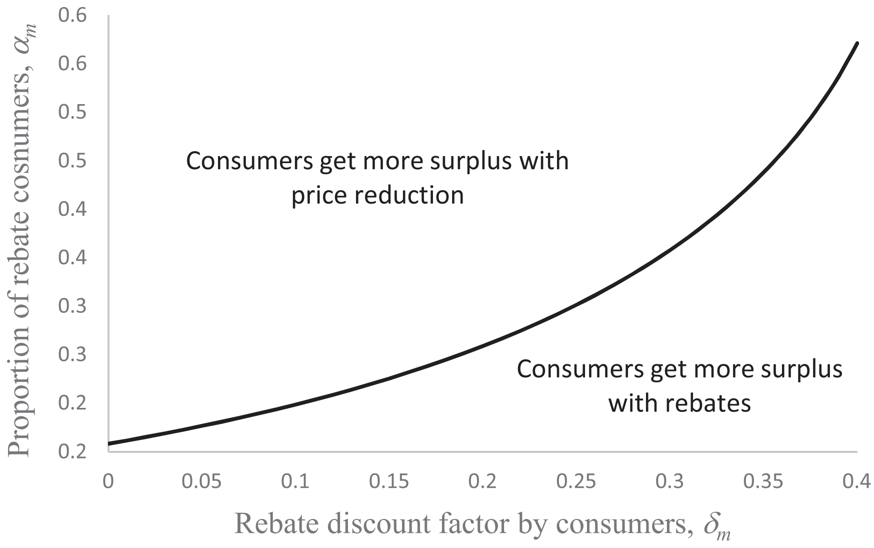

Fig. 6. Consumer surplus for rebates vs. price reductions.
$\rho_m = 0.6$.

segment is long, the retailer will not offer a price reduction if the price-insensitive segment is large. In this case, the retailer avoids having price-insensitive consumers get the price reduction during τ_d by not offering it.

With rebates, price-insensitive consumers do not put forth the effort to redeem rebates. Thus, τ does not affect profit and can be set to any value $0 < \tau < 1$. The retailer's profit is $\pi_m = \lambda_t + \lambda_s \psi \alpha_m (1 - \delta_m)(1 - \psi \rho_m)$. The retailer sets $p = 1$ to sell to all price-insensitive consumers. The optimal discount price, demand, profit and consumer surplus are obtained from the first order condition as $p_m^* = 1 - \frac{1}{2\rho_m}$, $D_d^* = \lambda_t + \frac{\alpha_m(1-\delta_m)\lambda_s}{2\rho_m}$, $\pi_m^* = \lambda_t + \frac{\alpha_m(1-\delta_m)\lambda_s}{4\rho_m}$, and $\chi_m^* = \frac{\alpha_m(1-\delta_m)\lambda_s(\delta_m+2\rho_m-1)}{8\rho_m^2}$, respectively.

Comparing the optimal discount amount for price reduction and optimal rebate value in the presence of a price-insensitive consumer segment, Corollary 2 follows.

Corollary 2. *If* $\rho_m < \bar{\rho}_m \equiv \frac{\alpha_m(1-\delta_m)\lambda_s(1-\lambda_t)}{(1-(1+\tau_d)\lambda_t)^2}$ *then*

1. *The retailer has a larger profit with rebates than with price reduction.*
2. *The demand is larger with rebates than with price reduction.*
3. *The retailer offers a larger reward (lower discount price) and sells more products with rebates than with price reduction.*
4. *If rebates are more profitable for the retailer than price reduction promotion, then consumer surplus will be smaller.*

The expression for $\bar{\rho}_m$ and observation 1 indicate that as the duration needed for price-sensitive consumers to purchase the product τ_d increases, rebates become more effective at larger redemption rates, i.e. larger $\bar{\rho}_m$. This is because price-insensitive consumers do not have to take any action to receive the price reduction during τ_d. Therefore, the performance of price reduction promotion deteriorates as τ_d increases. Furthermore, we can show that as λ_t increases on the interval $[0, 1]$, $\frac{\alpha_m(1-\delta_m)\lambda_s(1-\lambda_t)}{(1-(1+\tau_d)\lambda_t)^2}$ increases. Therefore, larger price-insensitive consumer segment makes rebates more profitable than price reduction promotion at larger redemption rate. Observations (2–3) indicate that if rebates are profitable for the retailer, then they will also have larger demand and lower discount price (for consumers who redeem them) than price reduction promotions. Observation 4 indicates that if rebates result in higher profit, then they will also result in lower consumer surplus.

4.3. Additional objectives of marketing promotions

In most models, the retailer is assumed to be risk-neutral profit-maximizing decision maker. However, in our discussion of marketing performance measures, it is clear that while profit is one of the most important measures, it is not the only one. Other measures such as market share, sales, customer service are important as well. Suppose that the retailer uses a multi-criteria objective function in which profit receives a weight of W_p, where $0 < W_p < 1$, and sales receive a weight of W_s, where $0 < W_s < 1$, and $W_p + W_s = 1$. Then, for price reduction, the objective function is to maximize $\pi_d = W_p (p_d (1 - p_d (1 - \psi_d + \psi_d^2))) + W_s(1 - p_d(1 + \psi_d))$. For rebates, the objective is to maximize $\pi_m = (1 - p_m)(p_m W_p + W_s) + p_m \psi_m \alpha_m (1 - \delta_m)(p_m W_p + W_s - p_m W_p \psi_m \rho_m)$. The optimal prices, maximum objective function value, demand, and consumer

Table 6
Measures of promotions performance with endogenous regular and discount prices.

	Price reduction	Cash mail-in rebates
Demand	$\frac{2}{3W_p}$	$\frac{2\rho_m}{W_p(4\rho_m-\alpha_m(1-\delta_m))}$
Objective function	$\frac{1}{3W_p}$	$\frac{\rho_m}{W_p(4\rho_m-\alpha_m(1-\delta_m))}$
Regular price	$\frac{2-3W_s}{3-3W_s}$	$\frac{\alpha_m(W_s-W_s\delta_m)+2(W_p-W_s)\rho_m}{W_p(4\rho_m-\alpha_m(1-\delta_m))}$
Discount price	$\frac{1}{2-3W_s}$	$\frac{W_s\alpha_m(1-\delta_m)+2(W_p-W_s)\rho_m-1}{W_p(4\rho_m-\alpha_m(1-\delta_m))}$
Consumer surplus	$\frac{1}{9(1-W_s)^2}$	$\frac{\alpha_m^2(1-\delta_m)^2-\alpha_m(1-\delta_m)(1-\delta_m+2\rho_m)+4\rho_m^2}{2W_p^2(4\rho_m-\alpha_m(1-\delta_m))^2}$
Threshold $\bar{W}_s$ for discount	$\frac{1}{3}$	$\frac{2\rho_m-1}{4\rho_m-\alpha_m(1-\delta_m)}$

surplus are shown in Table 6. The threshold W_s in the last row of the table, denoted $\bar{W}_s$ is the value above which the discount/rebate price will become equal to zero, i.e. no promotion is offered and a single low price is used. From the solutions in Table 6, Corollary 3 follows.

Corollary 3. *Suppose W_p is the weight assigned to profit and W_s is the weight assigned to sales, where $W_p + W_s = 1$, in the objective function. If $\rho_m < \alpha_m(1 - \delta_m)$ and $W_s < \frac{2\rho_m-1}{4\rho_m-\alpha_m(1-\delta_m)}$, then*

1. *Rebates will result in larger optimal objective function value than price reduction promotion.*
2. *Rebates will have larger regular price than price reduction promotion.*
3. *Rebates will have lower discounted price (for consumers who redeem the rebate) than price reduction promotion.*
4. *Rebates will have larger demand than price reduction promotion.*

As Corollary 3 and Fig. 7 indicate, consumers are better off as the weight placed on the sales objective increases because the retailer decreases both the regular and promotion prices for both rebates and price reduction promotions. As the weight assigned to the sales objective increases the retailer relies more on decreasing the promotion price with rebates (i.e. increasing the rebate value) than with price reduction. For the parameters in the figure, the retailer offers a “free with rebate” at a much lower sales weight of $W_s = 0.11$ compared to the price reduction promotion where the discount price at $W_s = 0.11$ is 0.25. Thus, adding a sales objective with larger weight may lead the retailer to further penalize consumers who intend to redeem the rebate at purchase but do not relative to consumers who redeem the rebate.

5. Parameter estimation

From the above discussion, the effectiveness of different promotions depends on a set of parameters which may be difficult to estimate in practice. In this section, we illustrate how estimates of these parameters can be obtained from data available to retailers. We assume that the retailer has only the transactional data which consumers see on a printed receipt and stored in the retailer's database. For a single product, an example of this information is shown in Table 7.

The only variable which is unavailable to the retailer in Table 7 is the purchasers' reservation prices. We assume that the retailer is able to obtain the distribution of the reservation prices using experimental methods or survey methods such as conjoint analysis (Ghoniem & Maddah, 2015). We assume uniformly distributed reservation prices while noting that the

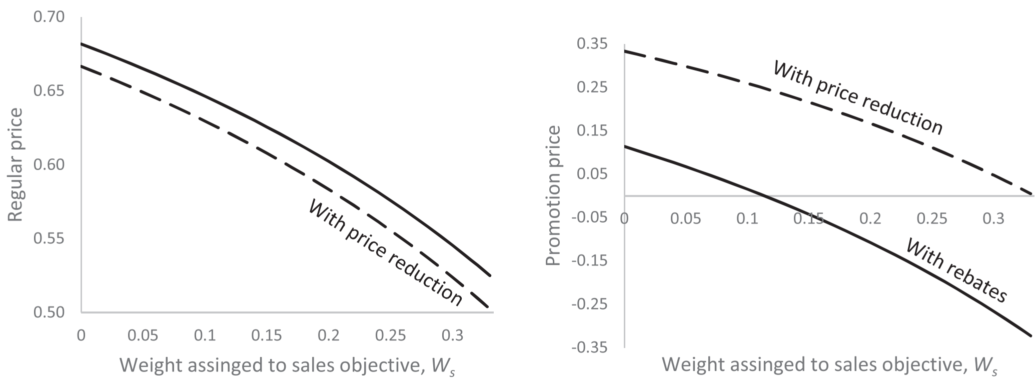

Fig. 7. Prices under rebates vs. price reductions.
$\rho_m = 0.6, \alpha_m = 0.8, \delta_m = 0.2$.

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Table 7
Retailer's available data.

	Date	Quantity	Regular price	Promotion type	Promotion price	Reservation price
Regular price period Duration = T	8/1/2017	1	40	None	40	43.1
	8/1/2017	1	40	None	40	47.3
	...	...	...	...	..	..
	...	...	...	...	..	..
	...	...	...	...	..	..
	...	...	...	...	..	..
	...	...	...	...	..	..
	10/30/2017	1	40	None	40	47.3
Promotion period Duration = τ	11/1/2017	1	40	Price reduction	32	
	11/1/2017	1	40	Price reduction	32	
	...	...	...	...	..	
	...	...	...	...	..	
	...	...	...	...	..	
	11/3/2017	1	40	Price reduction	32	
	11/4/2017	1	40	None	40	41.4

methodology can be modified to deal with other distributions. Since the goal in this section is parameter estimation, we no longer set $p = \frac{\omega}{2}$, $\lambda = 1$, or $\phi = 1$ and normalize the discount cycle to 1 period.

From the data in Table 7, the retailer can estimate the consumer arrival rate (λ). To estimate λ, we focus on the regular-price sales period of T days. For those consumers who purchased the product, the average reservation price is $\frac{\omega+b}{2}$. If N units are purchased over the regular sales period of T days, then $N = \lambda T \int_p^\omega \frac{1}{2b} du$ which gives

$$N = \bar{\lambda} T \frac{\omega - p}{2b} \tag{25}$$

Since N is available from the data, Eq. (25) gives the estimated average daily consumer arrival rate for all consumers of this product, including those who did not purchase, as

$$\bar{\lambda} = \frac{2Nb}{T(\omega - p)} \tag{26}$$

Now, suppose a price reduction of ψp is given to consumers over the promotion period of τ days, then the expected sales during the discount period of τ is

$$D_{d\tau} = \bar{\lambda}\tau \frac{\omega - p(1-\psi)}{2b} + \phi \bar{\lambda} T \frac{\psi p}{2b} = \frac{\bar{\lambda}}{2b}[\tau(\omega - p(1-\psi)) + \phi T \psi p] \tag{27}$$

Eq. (27) yields the following estimate of ϕ

$$\phi = \frac{2bD_{d\tau} - \lambda\tau(\omega - p(1-\psi))}{pT\lambda\psi} \tag{28}$$

Now suppose the retailer gives the same discount, i.e. ψp, using a coupon, cash mail-in rebate, or FGC, then the expected sales would be

$$D_{i\tau} = \alpha_i \bar{\lambda}\tau \frac{\omega - [p - (1-\delta_i)\psi p]}{2b} + (1-\alpha_i)\bar{\lambda}\tau \frac{\omega - p}{2b} + \bar{\lambda}\phi T \alpha_i \frac{(1-\delta_i)\psi p}{2b} \tag{29}$$

$$= \frac{\bar{\lambda}\left((\omega - p)\tau + \psi p(T\phi + \tau)\alpha_i (1-\delta_i)\right)}{2b}, \qquad i = n,\ g,\ or\ m \tag{30}$$

Eq. (30) can be rewritten as

$$\alpha_i (1-\delta_i) = \frac{2bD_{i\tau} - (\omega - p)\lambda\tau}{\psi p \lambda(\phi T + \tau)}, \qquad i = n,\ g,\ or\ m \tag{31}$$

The only unknowns in Eq. (31) are δ_i and α_i to be estimated from the data.

For the BOGO discount, the demand during τ is from the ϕ consumers with reservation prices satisfying $u - p < 0$ who arrived during T. These consumers have a demand of $2T\lambda\phi \int_{\frac{2(p-\psi p)}{1+\beta}}^{p} \frac{1}{2b} du$. Consumers with reservation prices satisfying $u - p < 0$

who arrived during τ behave the same way and their demand is $2\tau\lambda\int_{\frac{2(p-\psi p)}{1+\beta}}^{p}\frac{1}{2b}\,du$. Consumers with reservation prices satisfying $u-p>0$ who arrive during τ will buy two units and their demand is $2\tau\lambda\int_{p}^{\omega}\frac{1}{2b}\,du$. Thus, the total demand during τ is

$$D_{r\tau}=\frac{\lambda((1+\beta)\tau\omega+p(T\phi(\beta+2\psi-1)-2\tau(1-\psi)))}{b(1+\beta)}. \tag{32}$$

Since estimates of λ and ϕ are available, Eq. (32) yields the following estimate of β

$$\bar{\beta}=\frac{bD_{r\tau}+p\lambda(T+2\tau-2(T+\tau)\psi)-\lambda\tau\omega}{pT\lambda+\lambda\tau\omega-bD_{r\tau}} \tag{33}$$

5.1. The simulation

We now present the details of our simulation to examine the use of the above expressions for estimating the parameters. In this simulation, a full period is assumed to be 100 days, of which the promotion lasts for 5 days (i.e., days 96 through 100). We focus on a single product (e.g., jeans) with unit demand from each consumer. Each simulation run considers one of the five types of promotion (i.e., price reduction via discount, coupons, cash mail-in rebates, free gift cards, and BOGO discount). For each promotion type, the simulation run is replicated 100 times which is equivalent to the retailer having collected the data for 100 sales periods.

For each day of a simulation run, an arrival rate (λ) is generated. Each consumer has a reservation price. If the day is a non-promotion day, each arriving consumer computes her utility (computed from her reservation price, the offer price, and the discounting of her utility for the second item for BOGO) and makes a purchase if her reservation price is met. If the consumer does not purchase the item, then this consumer is considered part of the unmet demand and the consumer reservation price is saved. Thus, at the end of the non-promotion period, we have a list of consumers whose demand were not met at the regular price period with their reservation price. During the promotion period, the steps for newly arriving and pent-up demand consumers are different depending on the type of promotion. For each transaction, during the promotion and non-promotion period, we record the efficiency measures of the transaction, such as the consumer surplus, tax, and retailer profit. The simulation steps for each type of promotion during the promotion period are explained below.

For price reduction, each newly arriving consumer during the promotion computes her utility using the discounted price and purchases the item if her utility from purchase is positive. Otherwise, she does not purchase the item and the sale is lost. We then process the pent-up demand records accumulated for the first 95 days. We select only those transactions where the consumer's revised utility based on the discounted price is positive. For each of those transactions, the consumer purchases the item. The rest of the pent-up demand records are discarded.

For coupons, for each consumer a value for α is generated and compared against a threshold α_c to determine if the consumer incorporates the coupon discount into her purchase decision. If the consumer incorporates coupon discount into her purchase, then she computes her utility after discounting the face value of the coupon discount by δ_c. If her utility from purchase is positive, the consumer buys the item. If the consumer does not incorporate the coupon discount into her decision, then she behaves in the same way as during the non-promotion period.

For cash mail-in rebate and free gift cards, α and δ are handled similar to coupons. For BOGO discount a consumer arriving during the promotion and consumers in the pent-up demand face the decision of buying none, one, or two items. For each consumers the utility from the three choices are computed and the consumer chooses the action with the largest nonnegative utility.

All the parameters which need to be estimated are shown in Table 8. Table 8 also shows our estimates of these parameters. These estimates were obtained by using only the data outlined in Table 7 which is transactional data from only those consumers

Table 8
Parameter estimation.

	Parameters						
	λ	u	α*	δ*	$\alpha(1-\delta)$**	ϕ	β***
Distribution used in simulation	Poisson	U~ [30, 50]	U~ [0, 1]	U~ [0, 0.1]		NA	U~ [0.8, 1]
Mean	20	40	0.5	0.05	0.475		0.9
Estimated from data							
Average	20.01	NA			0.477	0.997	0.899
Minimum	19.94	NA			0.476	NA	NA
Maximum	20.06	NA			0.480		
Error	0.05%	NA			0.42%	−0.3%	−0.1%

* All promotions except Direct Discount promotion.
** Only coupon, cash mail-in rebate and free gift-card promotions.
*** Only incremental discount promotion.

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Fig. 8. Examples of BOGO.

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who made a purchase. As can be seen from the table, the estimates we are able to get from the transactional data are within $\pm 0.5\%$ from the true parameters. For some parameters, such as α and δ, we were able to only estimate $\alpha(1-\delta)$. For such parameters, it is sufficient for the retailer to invest some money to conduct surveys and or experiments to estimate one of the two parameters.

6. Discussion

Our results provide some interesting insights into marketing promotions. From the analysis in Section 3, we find that in promotions with immediate incentives, BOGO can be very effective when consumers' marginal utility from the second unit of the product is reasonably large and consumers do not stockpile. In our analysis we find that for reasonable parameters, if the utility from the second unit is around 0.71 of the utility from the first unit, BOGO is the most effective promotion among all five promotions along all effectiveness measures. These findings about BOGO are under the assumption of marginal unit cost of zero.

While data does not exist on the extent of use of BOGO among retailers, Fig. 8 illustrates BOGO's wide range of use on different products and by different retailers. These examples include different BOGO discounts, different products as well as services, and buy x units get y units free, e.g. "buy 2 get 3 free". For all these products the second unit is of considerable value to consumers. Even in the case of the vacations, many consumers may like to go in pairs and thus, the marginal utility from the second unit is large and BOGO is used. In Table 9 we show the number of BOGO offers for 6 retailers in different categories from their weekly ads. As can be seen, for some retailers, e.g., Pep Boys auto parts and Publix grocery chain, BOGO is almost as frequent as price reduction.

In promotions with delayed incentives, the use of FGC seems to be on the rise while rebates, in spite of their potential profitability, seem to be on the decline. Several reasons may have contributed to FGC's increased popularity. The first reason is the decrease in administrative costs of gift card programs. The time when gift certificates used to be frequently hand-written or printed is long gone as technology has greatly advanced. The second reason is that a \$1 gift card costs the retailer at most the cost of goods sold with the \$1, which may be considerably less than \$1. The third reason is that consumers may spend more than the value of the gift card and may purchase goods they would not have bought without the gift card. The fourth reason is that consumers have to return to the store or the website to redeem the card which increases store traffic. The fifth reason is that while both cash mail-in rebates and FGC are delayed incentives, gift cards can be usually redeemed anytime after purchase whereas rebates take several weeks to process. Given the present-bias of most consumers, a \$10 which can be redeemed after purchase may be more attractive than a \$10 rebate than may arrive in 8 weeks. For example, for a product priced at \$69.99, the perceived attractiveness of a \$10 instant rebate was the same as \$20 mail-in rebate (Kim, 2013). Finally, another aspect of gift cards that may contribute to their popularity is consumers' double mental accounting. Consumers often mentally deduct a gain, i.e. a free gift card, from perceived cost twice. A consumer may mentally deduct the value of the FGC from the price of the product as well as from the cost of the purchase when they redeem the gift card, which leads to more sales (Cheng & Cryder, 2018).

Similar to BOGO, data does not exist on the extent of use of FGC among retailers. However, Fig. 9 illustrates FGC's wide range of use on different products, including consumer durable and electronics, and by different retailers. In Table 9 we see that three retailers out of six were offering FGC in their weekly ads. We were not able to find any retailers currently offering their own rebates. We found only older rebate offers from two or more years ago as shown in Fig. 10. This is not surprising since many retailers such as Office Works in Australia and OfficeMax in the US have eliminated all their rebate programs (Currie & Mizerski, 2016).

Our analysis may shed some light on the reasons for the decline of rebate offers. An essential requirement for rebate offers is that many consumers take them into consideration in their mental accounting and do not significantly discount their value in their mental accounting. However, as many consumers had negative experience with rebates or heard of other consumers' negative experiences, they began to discard rebate offers or applied large discount factors to rebates. For rebates to be effective, retailers should begin by simplifying the redemption process and increasing consumers' confidence in the ability to redeem them without surprises. This will entail removing paper forms to be submitted and having an online submission option. The wait time between redemption and payment should be short. The degree to which this will increase redemption may be surprisingly small. Price-insensitive consumers may still find it not worthwhile to redeem the rebate and rebate's ability to price discriminate may still make them profitable. An interesting hyper promotion combination which has been introduced by some retailers is a FGC where only consumers who apply can receive it.

From the analysis in Section 4 we find that the existence of price-sensitive consumers may lead to a deterioration in the performance of price reduction promotions relative to FGC and cash mail-in rebates. The magnitude of this deterioration depends

Table 9
Number of promotions in weekly ads for some large retailers.

	Target	Publix	AutoZone	Pier 1 Imports	Pep Boys	J.C. Penney
Promotion dates	Nov. 11–17, 2018	Nov. 14–21, 2018	Nov. 13–Dec 21, 2018	Oct. 29–Nov. 26, 2018	Oct. 29–Nov. 25, 2018	Nov. 15–17, 2018
Buy one get one	5	60	1	6	26	5
Price reduction	104	86	22	40	30	122
Free gift cards	6	0	8	0	0	18
Cash mail-in rebate	0	0	0	0	0	0

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Fig. 9. Examples of FGC.

Fig. 10. Examples of cash mail-in rebates.

on the time duration required to have price-insensitive consumers buy the product. Improvements in information technology and the Internet are decreasing this duration and making the short “flash sales” to sell to price-insensitive consumers more common. We also find that when retailers place more importance on market share and sales volume, price reduction and BOGO promotions become more attractive.

7. Conclusion and future directions

We compared the performance of different types of promotions in terms of their ability to increase demand, profit, taxes, and consumer surplus. We found that when the utility from purchasing a second unit of the product is relatively large and consumers do not stockpile, BOGO promotions can be most effective in terms of increasing demand, profits, and consumer surplus, and possibly resulting in the highest taxes collected. If the marginal utility from the second unit diminishes significantly, then price reduction leads to the largest increase in sales, decrease in average price, and increase in consumer surplus. Both free gift cards and cash mail-in rebates have the potential to be best in terms of increasing profit and taxes. In the case of rebates, this increase in profits and taxes will occur at the expense of increased average price and reduced consumer surplus. In the case of FGC, this increase in profits and taxes may not require poor performance in terms of consumer surplus if the redemption rate is high and gift card cannibalization of regular sales is low.

We also found that when retailers find delayed incentives profitable, they offer deeper discounts and increase the regular price compared to an immediate incentive, resulting in increased consumer demand and profits but decreased consumer surplus. Consumers who redeem the promotion offer (i.e., rebates and FGC) are much better off with the delayed incentive but at the expense of those consumers who intended but failed to redeem the discount. We also found that the existence of price-insensitive consumer segment decreases the effectiveness of immediate incentive promotions.

Cash mail-in rebates, once popular among retailers and manufacturers, had strong potential for being profitable. However, consumers have come to dislike cash mail-in rebates for several reasons, as a review of their complaints on the Internet shows (McClure, 2012). Some customers think companies make the process for redeeming the rebates unnecessarily difficult with many steps. Another complaint is that redeemed rebates are often denied for arbitrary reasons (e.g., did not include UPC code, even when the customer confirms that correct UPC code was mailed). This resulted in many consumers ignoring the rebate offers altogether or significantly discounting the value of the rebate. As rebates declined in popularity, the use of FGC with purchase have become more common. This is because FGC have low administrative cost, they are easy to use for consumers, they result in lower cost for the retailer when redeemed, and may increase sales beyond their face value.

Our analysis can be extended in several ways. We analyzed a retailer's pricing decision without supply chain considerations. Future research can incorporate the manufacturer's response to the retailer's promotion by changing the wholesale price. This response may vary depending on the type of promotion. For example, the manufacturer may respond more favorably to BOGO promotions since it may have a higher positive effect on demand. We also dealt with deterministic demand of durable products. Another direction for future research is comparing promotions for perishable products which have stochastic demand. There products have declining consumer valuations over time and the ability of the retailer to sell excess inventory depends on the type of promotions used. Analyzing promotions in the context of competing retailers may provide another useful direction for future research. For example, among competing retailers, one may heavily rely on BOGO promotions, e.g. Publix grocery chain, while another may heavily rely on price reduction, e.g. Food Lion grocery chain.

Appendix A

Proof of Proposition 1.

1. For coupons, $\frac{\bar{p}_n}{\bar{p}_d} = \frac{(\psi+1)((1-\psi)\psi\alpha_n+1)}{(1-\psi)(2\psi+1)(\psi\alpha_n+1)}$ which is decreasing in α_n. Therefore, the minimum occurs at $\alpha_n = 1$ resulting in $\frac{\bar{p}_n}{\bar{p}_d} = \frac{-\psi^2+\psi+1}{-2\psi^2+\psi+1} > 1$ for $\psi > 1$ and thus the average price is higher with coupons.
2. For BOGO discount $\frac{\bar{p}_r}{\bar{p}_d} = \frac{(1+\psi)\left(3\beta+2(3-\beta)\psi-1-4\psi^2\right)}{(1-\psi)(1+2\psi)(3\beta+4\psi-1)}$, which is decreasing in β. Therefore, the minimum occurs at $\beta = 1$ resulting in $\frac{\bar{p}_r}{\bar{p}_d} = \frac{1+3\psi-2\psi^3}{1+3\psi-4\psi^3} > 1$ and thus the average price is higher with BOGO discounts.
3. For cash mail-in rebates and gift cards, $i = m, g$, $\frac{\bar{p}_i}{\bar{p}_d} = \frac{(1+\psi)(1+\psi\alpha_i(10\delta_i)(1-\psi\alpha_i\rho_i))}{(1-\psi)(1+2\psi)(1+\psi\alpha_i(1-\delta_i))}$, which is decreasing in α_i and ρ_i and increasing in δ_i. Therefore, the minimum occurs at $\alpha_i = \rho_i = 1$ and $\delta_i = 0$ resulting in $\frac{\bar{p}_i}{\bar{p}_d} = \frac{1+\psi-\psi^2}{1+\psi-2\psi^2} > 1$ and thus the average price is higher with cash mail-in rebates and gift cards. □

Proof of Proposition 2.

1. For BOGO discount, $\frac{D_r}{D_d} = \frac{3\beta+4\psi-1}{(\beta+1)(\psi+1)}$ which is increasing in β. Solving $\frac{D_r}{D_d} = \frac{3\beta+4\psi-1}{(\beta+1)(\psi+1)} = 1$ yields $\hat{\beta} = 3 - \frac{4}{2-\psi}$ above which BOGO discount will result in larger demand and below which price reduction will result in larger demand.
2. For coupons, $\frac{D_n}{D_d} = \frac{1+\psi\alpha_n}{1+\psi}$. Since $0 \le \alpha_n < 1$, $\frac{D_n}{D_d} < 1$ price reduction will result in larger demand than coupons.

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3. For cash mail-in rebates and gift cards, $i = m,g$, $\frac{D_i}{D_d} = \frac{1+\alpha_i(\psi-\psi\delta_i)}{1+\psi}$, which is increasing in α_i and decreasing in δ_i. Therefore, the maximum occurs at $\alpha_i = 1$ and $\delta_i = 0$ resulting in $\frac{D_i}{D_d} = 1$ and thus $\frac{D_i}{D_d} < 1$ for $0 < \alpha_i\delta_i < 1$ and price reduction has larger demand than cash mail-in-rebates and gift cards. □

Proof of Proposition 3.

1. For BOGO discount, $\frac{S_r}{S_d} = \frac{2(3+\beta-2\psi)(\beta+2\psi-1)}{(1+\beta)^2(1+\psi^2)}$ which is increasing in β for $\beta > \hat{\beta}$. solving $\frac{S_r}{S_d} = 1$ yields $\beta_s = \frac{1}{4}\left(4-\psi\left(8-\psi-\sqrt{16-(16-\psi)\psi}\right)\right)$ above which, BOGO discount has a higher consumer surplus and below which price reduction has higher consumer surplus.
2. For coupons, $\frac{S_n}{S_d} = \frac{1+\psi^2\alpha_n}{1+\psi^2}$. Since $0 \le \alpha_n < 1$, $\frac{S_n}{S_d} < 1$ price reduction will result in larger consumer surplus than coupons.
3. For mail-in rebates and gift cards, $i = m, g$, $\frac{S_i}{S_d} = \frac{1-\psi^2\alpha_i(1-\delta_i)(1-\delta_i-2\rho_i)}{1+\psi^2}$, which is decreasing in α_i and increasing in ρ_i and δ_i. Therefore, the maximum occurs at $\alpha_i = 0$ and $\delta_i = \rho_i = 1$ resulting in $\frac{S_i}{S_d} = \frac{1}{1+\psi^2} < 1$ and thus price reduction has larger consumer surplus than mail-in-rebates and gift cards. □

Proof of Proposition 4. The proof is presented in three parts

1. For BOGO discount vs. price reduction, $\frac{Z_r}{Z_d} = \frac{3\beta+2(3-\beta)\psi-1-4\psi^2}{(1+\beta)(1-(1-\psi)\psi)}$ which is increasing in β for $\beta > \hat{\beta}$. Solving $\frac{Z_r}{Z_d} = 1$ yields $\hat{\beta} = 3 - \frac{4}{2-\psi}$ above which BOGO discount will result in larger profit and below which price reduction will result in larger profit.
2. For $\beta \le \hat{\beta}$ price reduction has larger profit than BOGO discount. For cash mail-in-rebates, $\frac{Z_m}{Z_d} = \frac{1+\psi\alpha_m(1-\delta_m)(1-\psi\alpha_m\rho_m)}{1+(1-\psi)\psi}$ which is decreasing in ρ_m. Setting $\frac{Z_m}{Z_d} = 1$ yields $\hat{\rho}_m \equiv \frac{\psi+\alpha_m(1-\delta_m)-1}{\psi\alpha_m^2(1-\delta_m)}$ below which $\frac{Z_m}{Z_d} > 1$ and cash mail-in rebates yield larger profit. For gift cards, $\frac{Z_g}{Z_d} = \frac{1+\psi\alpha_g(1-\delta_g)(1+(R-(1-R+FR)\eta-1)\psi\alpha_g\rho_g)}{1+(1-\psi)\psi}$ which is decreasing in ρ_g. Setting $\frac{Z_g}{Z_d} = 1$ yields $\bar{\rho}_g \equiv \frac{\psi+\alpha_g(1-\delta_g)-1}{(1-R+(1-R-FR)\eta)\psi\alpha_g^2(1-\delta_g)}$ below which $\frac{Z_g}{Z_d} > 1$ and gift cards yield larger profit.
3. For $\beta > \hat{\beta}$ BOGO discount has larger profit than price reduction. For cash mail-in rebates, $\frac{Z_m}{Z_r} = \frac{\omega^2(1+\psi\alpha_m(1-\delta_m)(1-\psi\alpha_m\rho_m))}{8b\pi_r}$ which is decreasing in ρ_m. Setting $\frac{Z_m}{Z_r} = 1$ yields $\bar{\rho}_m \equiv \frac{(1+\beta)(1+\psi\alpha_m(1-\delta_m)(1-\psi\alpha_m\rho_m))}{3\beta+2(3-\beta)\psi-1-4\psi^2}$ below which $\frac{Z_m}{Z_r} > 1$ and cash mail-in rebates yield larger profit. For gift cards, $\frac{Z_g}{Z_r} = \frac{(1+\beta)(1+\psi\alpha_g(1-\delta_g)(1-(1-R+(1-R-FR)\eta)\psi\alpha_g\rho_g))}{3\beta+2(3-\beta)\psi-1-4\psi^2}$ which is decreasing in ρ_g. Setting $\frac{Z_g}{Z_r} = 1$ yields $\bar{\rho}_g \equiv \frac{2(1-\psi)(1-\beta-2\psi)+(1+\beta)\psi\alpha_g(1-\delta_g)}{(1+\beta)(1-R+(1-R-FR)\eta)\psi^2\alpha_g^2(1-\delta_g)}$ below which $\frac{Z_g}{Z_r} > 1$ and gift cards yield larger profit. □

Proof of Proposition 5. The proof is constructed using the same steps as Proposition 4. □

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